Multiobjetive Approach to Scheduling Problems in Airline Crew

Case Study of a Portuguese Airline

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Abstract

The airline industry is one of the major transportation industries that has contributed most to globalization. As such, there is a huge concern to make it increasingly profitable and sustainable. Therefore, due to the proportions that this industry has achieved, the airlines' concern with the service offered to it's customers tends to increase, leading to a need to re-evaluate the efficiency and speed with which everything is planned. The planning process is quite complex and when it isn't possible to accomplish a part of the process it is necessary to redo the whole thing or some parts or steps again and, in this case, the complexity increases because the time window is shorter as well as the available resources. Given the complexity and the logistical problems of working in an airline for the purpose of studying it's methods, for the thesis, the focus of the whole study was the planning of the crew. As we all know the human resources of any company are a fundamental part of the organization, one of the most important and therefore one of the ones that needs a greater investment. Consequently, a good planning and investment may lead the airline to distinguish itself from the competition both in commercial and in operational terms. As such a multiobjective optimization model was developed for the construction of schedule plans for an airline based in Portugal. This model has three objectives: the minimization of the extra cost, the balance of service hours and the balance of the flight hours between the same crew type. After the execution of the optimization model and the scenario analysis, it was possible to conclude that the number of crew members, relative to the initial data, can be reduced by 50% once the results are better.

Keywords: Schedules plan, airline company, aircrew, multiobjetive optimization model

1. Introduction

The airline industry is one of the major transportation industries that has contributed most to globalization. The size of this industry is causing more and more attention given to the service offered to it's customers. Therefore, to make the airline more profitable and sustainable, it is necessary to improve some aspects, namely the efficiency and speed with which everything is planned and evaluated. The service provided by the airlines offers much more than just the flight, it includes the planning of other small services such as the airplane, catering, flight crew, and cabin crew the planning of which occurs sequentially. If for some reason, internal or external to the airline, an unexpected event occurs near the operation in one of the stages of planning, the whole process will have to be planned again or at least most of it, which will inevitably result in a delay or flight cancellation.

The planning process is by itself quite complex. When it isn't possible to perform a part of it and a redo is needed, the complexity increases and the time in which to do it decreases, as well as the available resources. Flight planning as well as small services planning can be grouped into four main categories: aircraft, crew, landing and sales. However, due to the number of problems to be optimized in all four of these categories, the focus of the entire study will be only on the crew category. Since the human resources of any company, such as in this case the crew, are a fundamental part of organizations and one of the most important assets, and of those that need more investment.

Consequently, a good planning and investment can lead the airline to distinguish itself from the competition both commercially and operationally. Thus, the airline industry is one of the examples where the application of operational research methods and tools for resource planning is widely used.

As such, the main objective of this work is the development of a multiobjective optimization model for the construction of crew scales and to solve the main objective of this work, some intermediary goals must be accomplished:

- Understanding and describing the subject in relation to the airline industry;
- Analyze the current planning and re-planning processes used by airlines;
- Evaluate the state of the art with respect to optimization models for problems of crew scales;
- Develop a multiobjective optimization model for the problem under study;
- Apply the optimization model to a case study and analyze its results, with the goal of producing recommendations to the constructed model.

This paper will be structured as follows:

- Section 2: Problem Definition;
- Section 3: Literature Review;
- Section 4: Model;
- Section 5: Case Study;
- Section 6: Critical Results;
- Section 7: Conclusions;
- References;
- Annex.

2. Problem Definition

Of all the costs, the operational are the most prominent and a way to reduce them goes through the planning analysis of an air travel and its subsequent optimization. The fuel and labor costs represent a very significant part of operating costs. Thus, to combat fuel costs, airlines along with aircraft manufacturers try to develop more efficient engines and more economical forms of fuel. Regarding the reduction of labor costs, there is a need to optimize the use and allocation of personnel, (Ferreira *et al.,* 2011).

There are five possible optimizations for the crew. The first is related to workforce planning and should ensure that there are sufficient long-term aircrew to ensure the coverage of all future flights considering the training and vacation periods of all crews. The second is called crew pairing and consists of constructing a sequence of flight segments assigning generic crew to cover all flights, ensuring compliance with all governmental restrictions and labor agreements. The third, quite characteristic, is the assignment of crew which is often optimized along with pairing. The purpose is to allocate specific and available members to cover all the previously defined pairings and then form work schedules. These two, the pairing and the assignment, characterize the crew planning process. The fourth optimization is called crew control and aims to make small adjustments to the airport stops, to perform crew checks moments before the operation, to communicate last minute notifications and to confirm hotel accommodation. Finally, the crew has a stage of recovery in case any failure occurs. Corrective measures should be taken to guarantee the intended service of the customers. Thus, the purpose of this article is to efficiently solve the crew planning process, which, due to its multidimensional nature, results in the construction of a multiobjective optimization model. The model has three objectives: minimizing operating costs, balancing the number of service hours and balancing the number of flight hours for the flight and cabin crew. The flight crew is responsible for the take-offs, landings and the flight itself and the cabin crew, who according to APTCA - Associação Portuguesa de Tripulantes de Cabina -, is responsible for security of the passengers and the airplane, for rescue measures in case of an incident or accident during the flight and to service and ensure the comfort of the passengers during travel. Regarding crew schedules, in addition to the duration of the flight(s), also called flight time, the crew's working hours includes the presentation, pre-determined by the airline, before the flight, at the airport, the parking period at the airport of destination, the times of stop and the period of transport between the hotel and the airport (when necessary). The sum the time in which the crew is on duty is called the flight duty period.

3. Literature Review

Desaulniers et al. (1997) studied the problem of crew pairing. Aiming to find a solution that minimizes costs of pairing by ensuring coverage of all flight segments. The authors proposed a formulation through the flow problem in non-linear multiproduct networks. To solve the model, a partition and evaluation algorithm was used based on decomposing the Dantzig-Wolfe method. The main problem became a partition-type model of sets and the pairings were generated through short-path type subproblems. The results of the approximate approach were compared with the results of the systems used in Air France. Another approach to the same type of problem with the same objective and the use of the shortest path problem was studied and proposed by Muler *et al.* (2010). These authors formulated the model as a coverage problem and solved it by generating columns. Since the problem included aspects of robustness making the model resolution more complex, adaptations to the shortest path problem were necessary. The results of this approximate approach were obtained from actual data from Turkish local airlines.

A work with a different view from those previously mentioned was developed by Mohamed *et al.* (2016). For these authors, the pairing problem resolution depends on the aircraft rotation problem. With the objective of finding a solution that minimized the cost of the routes generated for each aircraft and the cost of the crew assigned to each flight, the authors proposed the resolution of an integrated model through a heuristic method divided into two approaches. First, an exact approach based on the resolution of an entire programming model was used, followed by a heuristic approach using a PSO (Particle Swarm Optimization) algorithm. To evaluate the performance of the approximate approach, a problem with local flight data in Malaysia has been solved. A recent study by Burak *et al* (2017) solves the crew pairing problem through a multiobjective approach. The authors proposed to minimize the operational costs through a mathematical programming modeling for the pairing problem, incorporating a representative portion of the fatigue. The problem of crew fatigue and alertness are modeled by the three alerts process model. In summary, the model deduces a trade-off between operating costs and fatigue levels. And the resolution was made through the columns generation where the resolution of the subproblem was using the shortest path problem with fatigue. Due to the numerous real data present for the analysis of exact results lagragian relaxation was used. Although there were few examples of solving the scheduling problem with a unique objective, it was possible to find in the literature two cases in which the authors had the task of solving a bi-objective problem. One of these cases was studied by Boufaied *et al.* (2015). The objectives, directed to the crew, consisted in: balancing the number of occurrences by destination and balancing the scales among the crew. First, the authors proposed solving the formulation in mathematical programming with the problem of generalized affectation and then for the optimization a meta-heuristic called a neighborhood variable research was used. The results of this approximate approach were obtained from actual data provided by TunisAir. Also following the previous example Teodorovic *et al.* (1998) studied the problem of crew scales. However, these authors have resorted to a single objective problem including two criteria which makes it a problem belonging to the multiobjective programming problems class. The proposed objective was to balance the workload among the crew. For this the authors proposed, the resolution through the diffuse control method and for the crew scales problem the heuristic method day-by-day. The results of this approximate approach were obtained through the use of real values of an airline.

There are some studies in the literature about integration problems. Both the integration of pairing problems with scheduling problems, as well as the integration of one of these with the problem of fleet allocation or aircraft rotation. Therefore, Cacchiani *et al.* (2013) studied the problems of fleet allocation, aircraft rotation and crew pairing. The objective of this integration was to minimize the cost through a weighted multi-criteria objective function. Then, the authors suggested a formulation in mathematical programming based on binary variables and described a generation of columns algorithm to obtain heuristic solutions. To analyze the results obtained, from this approximate approach, a comparison was made with the results of an airline that operates flights between the Canary Islands.

4. Model

The model is meant for Portuguese airlines and for short flight service periods, however it may also be applied to European airlines. In this case it would be necessary to know the limit values for the member state in which the airline was based and exchange the values. This is because, the model was based on Law no. 139/2004 of June 5, which in turn transposed the European Directive no. 2000/79/EC of 27 November. However, each member state can make changes to the limits without disrespecting the European Directive. However, only one part of the model is presented here and was simulated, mainly due to its dimensions and complexity.

The methodology applied to the model can be sequentially succinct in four singular stages: obtaining the pairing problem solutions of a specific flight type, establishing the input data and the parameters, executing the model and finally obtaining the set of optimal solutions that serve as the basis for the construction of Pareto curve graphs.

5. Case Study

To run the model below, the IBM ILOG CPLEX Optimization Studio software was used since it provides the optimal solution. As we can see the model is divided into seven sections, depending on the constraints characteristics. However, the presented mathematical model was programmed in the language recognized by the software.

In annex, table A.1, we can see the definition of each variable observed in the model.

A. Minimum of crew members number per airplane

$$\sum_{p=1}^{6} y_{pr}\, g_{pr} \geq 1 \qquad r \in R \tag{1}$$

$$\sum_{q=1}^{6} y_{qr}\, g_{qr} \geq 1 \qquad r \in R \tag{2}$$

$$\sum_{l=1}^{18} y_{lr}\, g_{lr} \geq 3 \qquad r \in R \tag{3}$$

$$\sum_{r=1}^{R} \sum_{i=1}^{I} \sum_{p=1}^{P} y_{pr}\, e_{pri} \leq 1 \tag{4}$$

$$\sum_{r=1}^{R} \sum_{i=1}^{I} \sum_{q=1}^{Q} y_{qr}\, e_{qri} \leq 1 \tag{5}$$

$$\sum_{r=1}^{R} \sum_{i=1}^{I} \sum_{l=1}^{L} y_{lr}\, e_{lri} \leq 1 \tag{6}$$

B. Balance of hours flown weekly for each type of crew member

$$\sum_{r=1}^{7}\left(\sum_{j=1}^{30}\left(\iota_{prj} - o_{prj}\right) + tvoo_p\right) - \sum_{r=1}^{7}\left(\sum_{j=1}^{30}\left(\iota_{p+1rj} - o_{p+1rj}\right) + tvoo_{p+1}\right) = (\alpha' - \alpha'') \quad p \in P \tag{7}$$

$$\sum_{r=1}^{7}\left(\sum_{j=1}^{30}\left(\iota_{qrj} - o_{qrj}\right) + tvoo_q\right) - \sum_{r=1}^{7}\left(\sum_{j=1}^{30}\left(\iota_{q+1rj} - o_{q+1rj}\right) + tvoo_{q+1}\right) = (\phi' - \phi'') \quad q \in Q \tag{8}$$

$$\sum_{r=1}^{7}\left(\sum_{j=1}^{30}\left(\iota_{lrj} - o_{lrj}\right) + tvoo_l\right) - \sum_{r=1}^{7}\left(\sum_{j=1}^{30}\left(\iota_{l+1rj} - o_{l+1rj}\right) + tvoo_{l+1}\right) = (\psi' - \psi'') \quad l \in L \tag{9}$$

$$\sum_{r=1}^{7}\left(\left(f_{pr} - b_{pr}\right) + tser_p\right) - \sum_{r=1}^{7}\left(\left(f_{p+1r} - b_{p+1r}\right) + tser_{p+1}\right) = (\alpha''' - \alpha'''') \qquad p \in P \tag{10}$$

$$\sum_{r=1}^{7}\left(\left(f_{qr} - b_{qr}\right) + tser_q\right) - \sum_{r=1}^{7}\left(\left(f_{q+1r} - b_{q+1r}\right) + tser_{q+1}\right) = (\phi''' - \phi'''') \qquad q \in Q \tag{11}$$

$$\sum_{r=1}^{7}\left(\left(f_{qr} - b_{qr}\right) + tser_l\right) - \sum_{r=1}^{7}\left(\left(f_{l+1r} - b_{l+1r}\right) + tser_{l+1}\right) = (\psi''' - \psi'''') \qquad l \in L \tag{12}$$

C. Maximum number of service hours with a pilot

$$\sum_{r=1}^{7}\left(\left(f_{prd} - b_{prd}\right)x_r y_{pr} e_{pri} p^{a}{}_{i} + vo_{pd}\right) \leq \mu_2(h^p, n^p)$$
$$h^p \in H^{P}\text{[1]},\ n^p \in N^P, p \in P,\ d \in D,\ i \in I \tag{13}$$

$$\sum_{r=1}^{7}\left(\left(f_{qrd} - b_{qrd}\right)x_r y_{qr} e_{qri} p^{a}{}_{i} + vo_{qd}\right) \leq \mu_2(h^p, n^p)$$
$$h^p \in H^P,\ n^p \in N^P,\ q \in Q,\ d \in D,\ i \in I \tag{14}$$

$$\sum_{r=1}^{7}\left(\left(f_{lrd} - b_{lrd}\right)x_r y_{lr} e_{lri} p^{a}{}_{i} + vo_{ld}\right) \leq \mu_1(h^a, n^a)$$
$$h^a \in H^A, n^a \in N^A,\ l \in L,\ d \in D,\ i \in I \tag{15}$$

D. Limits for total times of service with night flights

$$\sum_{r=1}^{7} \xi_r y_{pr} g_{pr} \leq 3 \qquad p \in P \tag{16}$$

$$\sum_{r=1}^{7} \xi_r y_{pr} g_{pr} \leq 3 \qquad q \in Q \tag{17}$$

$$\sum_{r=1}^{7} \xi_r y_{pr} g_{pr} \leq 3 \qquad l \in L \tag{18}$$

$$\left(\xi_{r+1} y_{pr+1} e_{pr+1i} f_{pr+1} + 36\right) \leq \left(y_{pr+2} e_{pr+2i} b_{pr+2}\right)$$
$$p \in P, r \in R, \xi_r\ y_p e_{pi} = 1, \xi_{r+1} y_p e_{pi} = 1 \tag{19}$$

$$\left(\xi_{r+1} y_{qr+1} e_{qr+1i} f_{qr+1} + 36\right) \leq \left(y_{qr+2} e_{qr+2i} b_{qr+2}\right)$$
$$q \in Q, r \in R, \xi_r y_q e_{qi} = 1, \xi_{r+1} y_q e_{qi} = 1 \tag{20}$$

$$(\xi_{r+1} y_{tr+1} e_{lr+1i} f_{lr+1} + 36) \leq (y_{lr+2} e_{lr+2i} b_{lr+2})$$
$$l \in L,\ r \in R, \xi_r y_l e_{li} = 1, \xi_{r+1} y_l e_{li} = 1 \tag{21}$$

$$\left(iniciop + des_p\right) y_p e_{pi} \leq \left(y_p e_{pi} b_{pr}\right)$$
$$p \in P, r \in R, \xi_{r-1} y_p e_{pi} = 1, \xi_{r-2} y_p e_{pi} = 1 \tag{22}$$

$$\left(iniciop + des_q\right) y_q e_{qi} \leq \left(y_q e_{qri} b_{qr}\right)$$
$$q \in Q, r \in R, \xi_{r-1} y_q e_{qi} = 1, \xi_{r-2} y_q e_{qi} = 1 \tag{23}$$

$$(iniciop + des_l) y_l e_{li} \leq (y_l e_{lri} b_{lr})$$
$$l \in L,\ r \in R, \xi_{r-1} y_l e_{li} = 1, \xi_{r-2} y_l e_{li} = 1 \tag{24}$$

E. Limits for total service and flight times

$$\sum_{r=1}^{7}\left(\sum_{j=1}^{30}\left(x_r\left(f_{pr} - b_{pr}\right) + vo_p\right)\right) \leq 55$$
$$p \in P,\ s \in S \tag{25}$$

$$\sum_{r=1}^{7}\left(\sum_{j=1}^{30}\left(x_r\left(f_{qr} - b_{qr}\right) + vo_q\right)\right) \leq 55$$
$$q \in Q, s \in S \tag{26}$$

$$\sum_{r=1}^{7}\left(\sum_{j=1}^{30}\left(x_r\left(f_{lr} - b_{lr}\right) + vo_l\right)\right) \leq 55$$
$$l \in L,\ s \in S \tag{27}$$

F. Other restrictions

$$prol_r x_r pa_i y_{pr} e_{pri} (fimr_r - iniciop_d) \begin{cases} \geq 0 \ \Rightarrow vo_{pd} = prol_r x_r pa_i y_{pr} e_{pri} (fimr_r - iniciop_d) \\ < 0 \ \Rightarrow vo_{pd} = 0 \end{cases} \tag{28}$$

$$prol_r x_r pa_i y_{qr} e_{qri} (fimr_r - iniciop_d) \begin{cases} \geq 0 \ \Rightarrow vo_{qd} = prol_r x_r pa_i y_{qr} e_{qri} (fimr_r - iniciop_d) \\ < 0 \ \Rightarrow vo_{qd} = 0 \end{cases} \tag{29}$$

$$prol_r x_r pa_i y_{lr} e_{lri} (fimr_r - iniciop_d) \begin{cases} \geq 0 \ \Rightarrow vo_{ld} = prol_r x_r pa_i y_{lr} e_{lri} (fimr_r - iniciop_d) \\ < 0 \ \Rightarrow vo_{ld} = 0 \end{cases} \tag{30}$$

$$vo_p = \sum_{d=1}^{D} vo_{pd} \qquad d \subset S \tag{31}$$

$$vo_q = \sum_{d=1}^{D} vo_{qd} \qquad d \subset S \tag{32}$$

$$vo_l = \sum_{d=1}^{D} vo_{ld} \qquad d \subset S \tag{33}$$

[1] Since the reference airline is about 80 times smaller than the Portuguese airline and there is only one airplane that requires 3 cabin crew, considering the capacity of passengers it can carry, the number of cabin crew used is 18 instead of 33.

[2] Of the 7 types of aircraft available in the fleet, the number of flight crew required is 1 pilot and 1 co-pilot for all.

$$tvoo_p = \sum_{d=1}^{D}(voo_{pd} + vo_{pd} + v_{pd} + vr_{pd} + vre_{pd}) \quad d \subset S \tag{34}$$

$$tvoo_q = \sum_{d=1}^{D}(voo_{qd} + vo_{qd} + v_{qd} + vr_{qd} + vre_{qd}) \quad d \subset S \tag{35}$$

$$tvoo_l = \sum_{d=1}^{D}(voo_{pd} + vo_{pd} + v_{pd} + vr_{pd} + vre_{pd}) \quad d \subset S \tag{36}$$

$$tser_p = \sum_{d=1}^{D}\left(prol_r y_{pr} e_{pri}(fimr_r - inicio_d)\right) \quad d \subset S \tag{37}$$

$$tser_q = \sum_{d=1}^{D}\left(prol_r y_{qr} e_{qri}(fimr_r - inicio_d)\right) \quad d \subset S \tag{38}$$

$$tser_l = \sum_{d=1}^{D}(prol_r y_{lr} e_{lri}(fimr_r - inicio_d)) \quad d \subset S \tag{39}$$

$$prol_r y_{pr} e_{pri}(fimr_r - inicio_d + 36) \begin{cases} \geq 0 \ \Rightarrow des_p = prol_r y_{pr} e_{pri}(fimr_r - inicio_d + 36) \\ < 0 \ \Rightarrow des_p = 0 \end{cases} \tag{40}$$

$$prol_r y_{pr} e_{pri}(fimr_r - inicio_d + 36) \begin{cases} \geq 0 \ \Rightarrow des_p = prol_r y_{pr} e_{pri}(fimr_r - inicio_d + 36) \\ < 0 \ \Rightarrow des_p = 0 \end{cases} \tag{41}$$

$$prol_r y_{pr} e_{pri} \begin{cases} \geq 0 \ \Rightarrow des_p = prol_r y_{pr} e_{pri}(fimr_r - inicio_d + 36) \\ < 0 \ \Rightarrow des_p = 0 \end{cases} \tag{42}$$

G. Objetive Functions

$$Minimize \quad z_1 = \sum_{p=1}^{P}\sum_{r=1}^{R}\left(\sum_{j=1}^{J}\left(\Phi_p\left(\iota_{prj} - o_{prj}\right)\right)\right) + \sum_{q=1}^{Q}\sum_{r=1}^{R}\left(\sum_{j=1}^{J}\left(\Phi_q\left(\iota_{qrj} - o_{qrj}\right)\right)\right) + \sum_{l=1}^{L}\sum_{r=1}^{R}\left(\sum_{j=1}^{J}\left(\Phi_l\left(\iota_{lrj} - o_{lrj}\right)\right)\right) \tag{43}$$

$$Minimize \quad z_2 = (\alpha' + \alpha'') + (\phi' + \phi'') + (\psi' + \psi'') \tag{44}$$

$$Minimize \quad z_3 = (\alpha''' + \alpha'''') + (\phi''' + \phi'''') + (\psi''' + \psi'''') \tag{45}$$

To apply the above model, it is necessary to define some characteristics about the airline that will serve as reference. And as a way for the reference airline to be as reliable as possible its values will be about 80% smaller than a Portuguese airline. The table 1 lists the characteristic values of the two airlines.

For the execution of this model all units of time were converted in minutes to simplify and avoid any conflicts. Thus, table 2 summarizes all the parameters in which its designation, its value and its definition can be verified.

Table 1 - Values of a Portuguese airline *versus* a reference airline

	Portuguese airline	Reference airline
No. of flight crew	939	6 pilots; 6 co-pilots
No. of cabin crew	2588	18[1]
No. of airplanes	80	1
Type of airplanes	7	1
Capacity by type of airplane	274; 263; 200; 162; 132; 106; 70	132 passengers
No. of flight crew required by type of airplane	1 pilot[2]; 1 co-pilot[2]	1 pilots; 1 co-pilots
No. of cabin crew required by type of aircraft	6; 6; 4; 4; 3; 3; 2	3

Table 2 - Values assigned to time parameters

Parameter	Value [min]	Definition
minTotais	10800	Minutes corresponding to one week
descansoEntreRotas	2160	Required minutes of rest after two consecutive night flights
maxVooSemanal	3300	Maximum flight minutes that crew members may carry out
inicioProximaSemana	10080	Exact minute starting next week

In addition to the basic salary that the crew members receive, it is quite common for some airlines to pay an extra value per hour flown, table 3. Thus, in the referenced airline all the crew receive an extra value per minute flown, since the whole model is converted in minutes. These values are assumed and are different depending on the type of crew member and their responsibilities.

Table 3 - Extra values assigned to each crew member by the minutes flown

Parameter	Value [€/min]	Definition
extraPiloto	1.40	Extra value received by the pilot for each minute flown
extraCopiloto	0.70	Extra value received by the co-pilot for each minute flown
extraTCabina	0.35	Extra value received by the cabin crew member for each minute flown

For the crew members to be properly assigned to PSVs, the knowledge of some characteristics and data is required in a timely manner, annex - table A.2. This data, taken from a website, is from a Portuguese airline and only one type of specific aircraft, the Airbus A319-111 with autopilot.

For entry data, none of the crew had hours of rest or flight that lasted from the previous week to the week to simulate.

6. Critical Results

Given the software limitations it was essential to transform the objective functions into one. In this way, the merging of the three objective functions has given rise to a function similar to the one represented in (46) where each μ represents a coefficient by which each objective function is valued.

$$f = min\left(\mu_1 f_1(x) + \mu_2 f_2(x) + \mu_3 f_3(x)\right) \qquad (46)$$

To construct the Pareto curve, the value of the coefficients was varied and after the model was executed for the different values of the coefficients, the solutions of figure 1 were obtained. With the first image, it is possible to visualize the location where the three straight lines are closest and to understand that when higher coefficients are assigned to the first objective function, the solutions are also higher, when the values of the coefficients are all different from zero.

Thus, the points presented in figure 2, outline the Pareto curve, since they constitute the set of non-dominated solutions in minimizing the objective functions, except for the black point (0,0,0) that serves as a graphic reference. A non-dominated solution is characterized by the fact that in the search space Z there is no other feasible solution that simultaneously improves all the objectives.

From table 4, which represents the nineteen non-dominated and Pareto front points, it is possible to conclude from an initial analysis that the point denoted as "point 1" reaches the lowest total z value. This is because the coefficient of the first function has a value of one and the remainder has a null value, this means that the model is only concerned with assigning the minimum number of crew members required to respect the first three constraints and in this case the value of z is only calculated by cost / minute of the minutes flown.

The same does not happen with the "point 19" that of the non-dominated points is what has a higher total z value. This is due to the difficulty of the model to carry out the assignment of crew members, minimizing in a very similar way the costs/minute of the time flown, the difference of minutes flown between crew members and also the difference of minutes of service performed, since the coefficients have very approximate values.

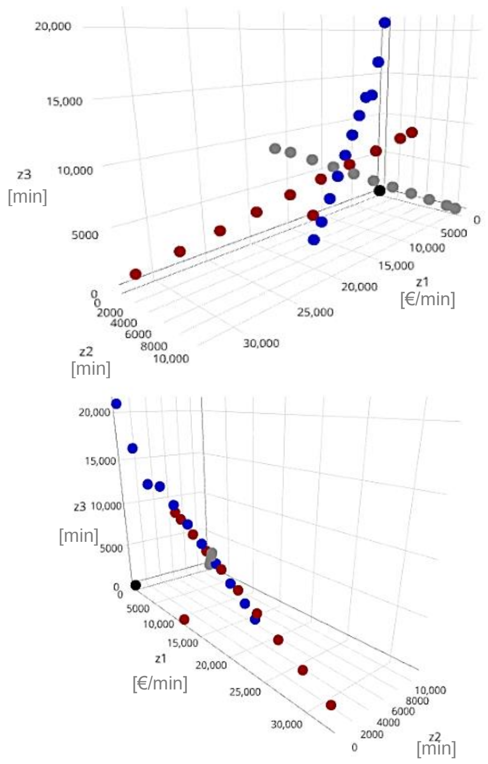

Figure 1 – Set of solutions

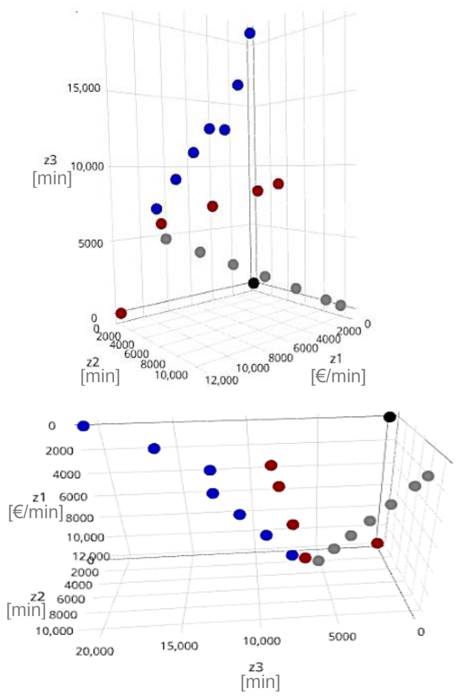

Figure 2 – Non-dominated solutions

Analyzing in more detail the obtained results show that the higher the coefficients of the differences, that is, the more minutes flown by the same crew and the greater the proximity in terms of value between the coefficients, the higher the final values of z are.

Table 4 – Non-dominated solutions characterization

Point	μ_1	μ_2	μ_3	z1 [€/min]	z2 [min]	z3 [min]
1	1	0	0	11793,6	0	0
2	0,30	0,35	0,35	10614,2	3931,2	6394,5
3	0,20	0,40	0,40	7076,16	4492,80	7308,00
4	0,10	0,45	0,45	3538,08	5054,40	8221,50
5	0,05	0,475	0,475	1769,04	5335,20	8678,25
6	0,025	0,95	0,025	884,52	10670,4	456,75
7	0,05	0,90	0,05	1769,04	10108,8	913,50
8	0,20	0,80	0,20	3538,08	8985,60	1827,00
9	0,15	0,70	0,15	5307,12	7862,40	2740,50
10	0,20	0,60	0,20	7076,16	6739,20	3654,00
11	0,25	0,50	0,25	8845,20	5616,00	4557,50
12	0,30	0,40	0,30	10614,2	4492,80	5481,00
13	0	0	1	0	0	20900,0
14	0,05	0,05	0,90	1769,04	561,60	16443,0
15	0,10	0,10	0,80	3538,08	1123,20	12789,0
16	0,15	0,15	0,70	5307,12	1684,80	12747,0
17	0,20	0,20	0,60	7076,16	2246,40	10962,0
18	0,25	0,25	0,50	8845,20	2808,00	9135,00
19	0,30	0,30	0,40	10614,2	3369,60	7308,00

Other conclusions that can be drawn from the results are that for all crew, the flights ended the week they started, so there were no flights that went from a week to another or minutes to rest.

As a way of confirming, we can see the table in Annex 1 where all PSV features are found. In this case, the last flight of the last PSV was between Lisbon (LIS) and Munich (MUC) and it landed at 23:20. There was no need to accumulate flight minutes for the following week. The same is true of the obligatory rest minutes for night flights, since the last flights were carried out in daytime and non-night time. Hence the values of the variables $cumulDescansoPi$,$cumulDescansoQi$ e $cumulDescansoTi$ be "Falso".

Finally, it is relevant to analyze the data in table 5 since it is the results of the assignments of points "1", "2" and "13" and it is through these that the weekly work scales are constructed. It is possible to observe that the "point 1" in comparison to the others is quite different, because in this case the one that has more value for the model is to minimize the costs and in this way it assigns only one pilot, a co-pilot and three cabin crew since they are the minimum required. Regarding the choice of crew, that is unimportant, since it is not necessary that they work and fly the same number of minutes. On the other hand, when the values of the remaining objective functions increase, the model chooses to select the crew members in teams of 3, in the case of pilots and co-pilots, and 9 in the case of cabin crew. This is because the model is pressed so that the number of flight and service hours is as similar as possible between crew members of the same gender. If "point 2" is analyzed in more detail and if the number of flight hours that Pilot 1, 3 and 5 are calculated compared to pilots 2, 4 and 6 is determined, the number of hours flown by the first pilots is about 36 hours and 31 minutes while the second ones fly 25 hours and 53 minutes and regarding hours of service, the first ones total about 51 hours and 55 minutes while the second ones only amount to 46 hours and 5 minutes. In fact, given the similarity between the values of the coefficients it was expected that the number of hours flown and service hours would be more approximate, however it should be noted that the allocation is made difficult due to the few options of choice. Another point that stands out is the "point 13", as it is distinguished by the distributions of crew members. In this case, the model does not choose to combine the crew members into teams, but allocates them in such a way that the number of hours of service is as similar as possible, since it is only this factor that interests them, $\mu_3 = 1$ - table 4.

After the previous analysis it became relevant to understand how the results would be affected by making small changes in the data. In this way, the number of crew members was reduced by 50% - scenario 1.

Compared to the graphs in Figure 2, the distribution of the points is similar, but the range of variation in z1 is significantly lower, which means that the final solution is also smaller. Of all the non-dominated points, the solution that reaches the best value is again the "point 1", since the sole objective of the model is to minimize the costs not worrying, therefore, as the conjunction of the crew to the flight and service time is as similar as possible. This value compared to the initial model becomes equal since the number of crew assigned is the same for the seven PSV as well as the values received by the crew by the minutes flown. This means that even though the number of crew members has been reduced by 50%, the flight time accumulated by them are higher, fulfilling all the restrictions imposed. On the other hand, the highest value of z is that of "point 23" characterized by having the highest coefficient of z3. This point, compared to the point with greater value of the initial model has a difference of 5570.05 points, representing a reduction of about 16.8%. Regarding the assignment of the crew to the work schedules, as can be seen in table 6, all non-dominated points have the same combination of crew members except point 1.

Table 5 – Characterization of non-dominated solutions for points 1, 2 and 13

Point	PSV	P attributed	Q attributed	T attributed
1	1	1	1	1, 2, 3
	2	6	6	14, 16, 18
	3	5	5	12, 13, 18
	4	5	5	1, 12, 13
	5	1	1	8, 9, 14
	6	1	1	1, 2, 3
	7	1	1	1, 2, 3
2	1	1, 3, 5	2, 4, 6	1, 3, 5, 7, 9, 11, 13, 15, 17
	2	1, 3, 5	2, 4, 6	1, 3, 5, 7, 9, 11, 13, 15, 17
	3	2, 4, 6	1, 3, 5	2, 4, 6, 8, 10, 12, 14, 16, 18
	4	2, 4, 6	1, 3, 5	1, 3, 5, 7, 9, 11, 13, 15, 17
	5	1, 3, 5	2, 4, 6	1, 3, 5, 7, 9, 11, 13, 15, 17
	6	2, 4, 6	2, 4, 6	2, 4, 6, 8, 10, 12, 14, 16, 18
	7	1, 3, 5	2, 4, 6	1, 3, 5, 7, 9, 11, 13, 15, 17
13	1	1,2,3,4,5,6	1,2,3,4,5,6	1, 3, 5, 7, 9, 11, 13, 15, 17
	2	1,2,3,4,5,6	1,2,3,4,5,6	2, 4, 6, 8, 10, 12, 14, 16, 18
	3	2, 5	2, 5	4, 6, 10, 12, 16, 18
	4	1,3,4,6	1,3,4,6	2, 8, 14
	5	2,5	2,5	4, 6, 10, 12, 16, 18
	6	1, 3, 4, 6	1, 3, 4, 6	2, 8, 14
	7	1,2,3,4,5,6	1,2,3,4,5,6	1, 3, 5, 7, 9, 11, 13, 15, 17

Table 6 – Characterization of the non-dominated solutions (scenario 1)

Point	PSV	P attributed	Q attributed	T attributed
1	1	1	1	1, 2, 3
	2	3	3	5, 8, 9
	3	2	2	6, 7, 9
	4	2	2	1, 6, 7
	5	1	1	3, 4, 5
	6	1	1	1, 2, 3
	7	1	1	1, 2, 3
[2,…,32]	1	2	2	2, 4, 6, 8
	2	2	2	5, 8, 9
	3	1, 3	1, 3	1, 3, 5, 7, 9
	4	2	2	2, 4, 6, 8
	5	2	2	2, 4, 6, 8
	6	2	2	2, 4, 6, 8
	7	2	2	2, 4, 6, 8

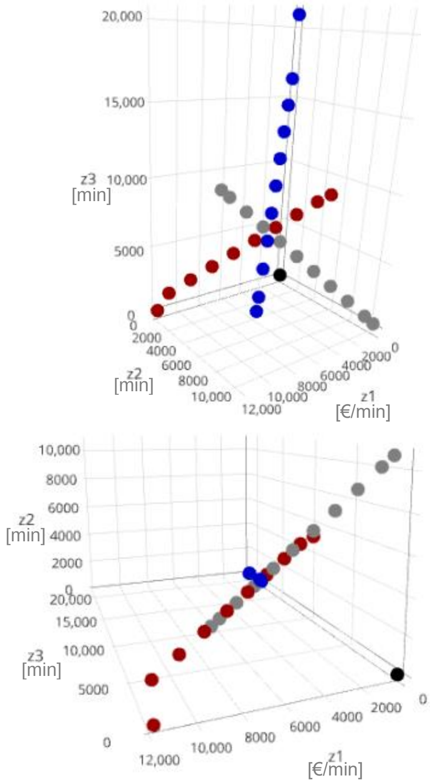

Figure 3 – Non-dominated solutions (scenario 1)

7. Conclusions

Relatively to obtain the results and subsequent analysis, the coefficients associated to each objective function were varied decimally and it should be noted that all the obtained results are optimal solutions that consider the compliance of all constraints and the value given to the coefficient of each objective function. To analyze the impact on the results, a scenario was analyzed which consisted in reducing the number of crew members by 50%. Obtaining the values gave greater attention to the non-dominated results, since the minimums were obtained and with this analysis of results it was possible to conclude that better values are obtained in the minimization of the objectives when the number of crew members is reduced by half.

In a future work it would be important to first fully simulate the initial model formulated with as many PSV and crew as possible so that the reference airline becomes more reliable as well as the solutions obtained. Another important aspect would be to keep the three objective functions individualized and not in a single objective function composed of the three.

Secondly, it would be relevant to generalize the initial model for all flights, whether short, medium or long. This generalization will change the temporal aspects, in particular regarding PSV prolongations, since the duration may extend to more than two days. By norm and as it was verified with the case study the PSV were associated to one day and in some cases to carry over to the next day, nevertheless its duration never exceeded twenty four hours. However, in long-term PSV, the difference between the end of the PSV and the beginning may be a few days and due to these cases, the restrictions should be modified.

Finally, related to one of the most important aspects for any organization - human resources - it would be interesting to add to the data and the model the preferences of the crew, namely the duration of the PSV (short, medium or long) and the preferred destinations. Another equally important aspect related to human resources would be to characterize the cabin crew members by the different extents as well as for the flight crew where they are distinguished between pilots and co-pilots.

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Annex

Table A.1 – Sets, indexes, parameters, decision variables and binary variables defined for the optimization model

$\boldsymbol{P = \{1, \dots, p, \dots, \bar{p}\}}$		means the set of pilots;
$\boldsymbol{Q = \{1, \dots, q, \dots, \bar{q}\}}$		means the set of co-pilots;
$\boldsymbol{L = \{1, \dots, l, \dots, \bar{l}\}}$		means the set of cabin crew;
$\boldsymbol{R = \{1, \dots, r, \dots, \bar{r}\}}$		means the set of PSV;
$\boldsymbol{N^P = \{1, \dots, n^p, \dots, 4\}}$		means the set of number of aircraft for flight crew with one pilot;
$\boldsymbol{H^P = \{1, \dots, h^p, \dots, 8\}}$		means the set of hours of presentation for crew with a pilot;
$\boldsymbol{I = \{1, \dots, i, \dots, \bar{\imath}\}}$		means the set of aircraft;
$\boldsymbol{J = \{1, \dots, j, \dots, \bar{\jmath}\}\,;\, J \subset R}$		means the set of flight segments associated with PSV;
$\boldsymbol{S = \{1\}; S}$		means the set of seven consecutive days - week;
$\boldsymbol{D = \{1, \dots, d, \dots, 7\}; D \subset S}$		means the set of twenty-four consecutive hours - day;
$\boldsymbol{b_{prd}, f_{prd}, b_{pr}, f_{pr}; b_{qrd}, f_{qrd}, b_{qr}, f_{qr}; b_{lrd}, f_{lrd}, b_{lrs}, b_{lr}, f_{lr};}$	$p \in P, q \in Q, l \in L, d \in D, r \in R$	means the start and end of PSV r for its crew, in periods of time n, s, d, t, and annually. The start of the PSV begins with the presentation of the crew member at the base;
$\boldsymbol{m_{rj}, n_{rj}}$	$r \in R, j \in J$	means the time of departure and arrival at the base, respectively;
$\boldsymbol{o_{rj}, \iota_{rj}}$	$r \in R, j \in J$	means the time of departure and arrival at the destination;
$\boldsymbol{o_{prj}, \iota_{prj};\ o_{qrj}, \iota_{qrj}; o_{lrj}, \iota_{lrj}}$	$p \in P, q \in Q, l \in L, r \in R, j \in J$	means the time of departure and arrival at the destination for the respective crew;
$\boldsymbol{\gamma_{pr}, \varrho_{pr}; \gamma_{qr}, \varrho_{qr}; \gamma_{lr}, \varrho_{lr}}$	$p \in P, q \in Q, l \in L, r \in R$	means the start and end of the interval for the respective crew member;
$\boldsymbol{vo_{pd}, vo_p; vo_{qd}, vo_q; vo_{ld}, vo_l;}$	$p \in P, q \in Q, l \in L, d \in D$	means the extension time when the subtype of flight has only one pilot in command of the airplane;
$\boldsymbol{tvoo_p, tvoo_q; tvoo_l}$	$p \in P, q \in Q, l \in L$	means the extension time related to flight hours;
$\boldsymbol{tser_p, tser_q; tser_l}$	$p \in P, q \in Q, l \in L$	means the service extension time;
$\boldsymbol{iniciop_d;\ fimr_r, des_p, des_q\ des_l}$	$d \in D, r \in R, p \in P, q \in Q, l \in L$	means the beginning of the period of the end of the PSV and the rest to be carried out by the respective crew member;
$\boldsymbol{h_i}$	$i \in I$	means the total capacity of passengers that the airplane *i* supports;
$\boldsymbol{\upsilon_i}$	$i \in I$	means the minimum number of pilots required for airplane *i*;
$\boldsymbol{\chi_i}$	$i \in I$	means the minimum number of co-pilots required for airplane *i*;
$\boldsymbol{s_r}$	$r \in R$	means the number of flight segments after the interval;
$\boldsymbol{\Phi_p, \Phi_q, \Phi_l}$	$p \in P, q \in Q, l \in L$	means the extra value received by the respective crew member for the hours flown.
$\boldsymbol{\mu_2}$		means the function by which the limit of the number of hours of service is obtained for when there is a pilot in the same PSV;
$\boldsymbol{\alpha', \alpha'';\ \phi', \phi''; \psi', \psi'';}$		means the variables that allow the balance of the annual workload among the respective crew members;
$\boldsymbol{\alpha''', \alpha''''; \phi''', \phi''''; \psi''', \psi'''''};$		means the variables that allow the balance of the annual flight service among its crew members
$\boldsymbol{e_{pri}; e_{qri}; e_{lri}}$	$p \in P, q \in Q, l \in L, r \in R,\ i \in I$	1 if the crew member is qualified for PSV and for airplane *i*, 0 otherwise;
$\boldsymbol{y_{pr}; y_{qr};\ y_{lr}}$	$p \in P, q \in Q, l \in L, r \in R$	1 if the crew member is assigned to PSV, 0 otherwise;
$\boldsymbol{\beta_r}$	$r \in R$	1 if the crew member is performing a normal duty period, 0 otherwise;
$\boldsymbol{x_r}$	$r \in R$	1 if in PSV there is only one pilot, 0 otherwise;
$\boldsymbol{\xi_r}$	$r \in R$	1 if the PSV is nocturnal, 0 otherwise;
$\boldsymbol{prol_r}$	$r \in R$	1 if the extension occurs, 0 otherwise.
$\boldsymbol{p^a{}_i}$	$i \in I$	1 if the airplane has an autopilot, 0 otherwise.

Table A.2 – Sets, indexes, parameters, decision variables and binary variables defined for the optimization model

PSV	Start of PSV	End of PSV	PSV duration	Departure	Arrival	Departure time	Arrival time	Flight duration	Tyoe of PSV
1	12:05	21:50	10:15	Verona	Lisbon	12:35	12:38	07:51	normal
				Lisbon	Munich	14:55	18:55		
				Munich	Lisbon	19:40	21:50		
2	07:00	23:45	16:15	Lisbon	Venice	07:30	11:25	11:32	nocturnal
				Venice	Lisbon	12:15	14:20		
				Lisbon	London	15:05	17:45		
				London	Lisbon	18:45	21:20		
				Lisbon	Faro	23:05	23:45		
3	05:35	22:20	15:15	Faro	Lisbon	06:05	06:50	08:26	nocturnal
				Lisbon	Rome	09:00	12:50		
				Rome	Lisbon	13:35	15:40		
				Lisbon	Frankfurt	18:10	22:20		
4	05:45	23:00	17:45	Frankfurt	Lisbon	06:15	08:25	09:55	nocturnal
				Lisbon	Zurich	09:20	13:05		
				Zurich	Lisbon	13:55	15:45		
				Lisbon	Madrid	17:15	19:30		
				Madrid	Lisbon	20:15	20:30		
				Lisbon	Porto	22:00	23:00		
5	06:00	22:05	15:35	Porto	Lisbon)	06:30	07:30	09:59	normal
				Lisbon	Frankfurt	08:25	12:35		
				Frankfurt	Lisbon	13:25	15:35		
				Lisbon	Paris	16:25	19:50		
				Paris	Lisbon	20:40	22:05		
6	07:15	20:50	13:05	Lisbon	Funchal	07:45	09:30	07:32	normal
				Funchal	Lisbon	10:15	11:30		
				Lisbon	Luxembourg	14:40	18:20		
				Luxembourg	Lisbon	19:05	20:50		
7	12:00	23:20	09:50	Lisbon	Paris	12:30	15:55	07:09	normal
				Paris	Lisbon	16:40	18:05		
				Lisbon	Munich	19:20	23:20		